

Gaussian Mixture Initialization in Passive Tracking Applications

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Abstract – *This paper describes the approximation of a nonlinear posterior density by a Gaussian Mixture (GM). The GM is used to initialize a bank of Kalman filters. For each Gaussian term, a Kalman filter is started. The basic conditions and the quality of the approximation are discussed. Examples from different tracking applications, the multistatic tracking and passive emitter localization using TDOA measurements, are investigated. The results are discussed and compared with existing approaches. The RMS error of the estimate is used as an evaluation criterion. The performance of the Gaussian Mixture approach is analyzed in Monte Carlo simulations.*

Keywords: Gaussian Mixture, tracking, multistatic, TDOA, Kalman filter.

1 Introduction

This paper investigates two different types of passive tracking applications. First, in Multistatic Passive Radar the signal of illuminators of opportunity (like terrestrial television or radio antennas) are exploited to measure the reflected signal of an air target at a separate receiver, see for example [1]. Secondly, in passive emitter localization the target itself is emitting a signal which is obtained by a pair of sensors. In both applications we measure the time when the signal arrives at the receiver(s).

In passive radar applications, we are often faced with incomplete measurement information, which means that we are not able to estimate our full target state from measurements of a single source and receiver. Even though the Kalman Filter algorithm has been shown to provide a quite a robust solution for fusing measurements of different time steps in many applications, the quality of the resulting estimate is strongly dependent on a good initialization [2].

Difficulties appear if the approximation of the initial Cartesian covariance matrix by a single Gaussian fails. A similar effect has also been noted for other

tracking scenarios (see for example [3]) if the sensor provides only poor azimuth measurements. With increasing azimuth inaccuracy and increasing distance to the sensor, the Cartesian uncertainty region develops more and more to a crescent shape. If we approximate this by a single ellipse (approximation by single Gaussian), the ellipse must either be very large or ignores possible target states.

This paper discusses the initialization problems of the two introduced tracking applications and analyzes the use of a GM approach. We will give numerical results analyzing the Cartesian estimation performance in terms of the root mean squared error and comparison to the Cramér Rao Lower Bound (CRLB).

The paper is organized as follows: after this introduction, we will describe the passive tracking scenarios in section 2. Section 3 provides some fundamentals on GM. Applications to passive tracking and results are given in 4. Section 5 concludes this paper.

2 Scenario Description

In this section the two types passive radar are described and the application of GM approximation is motivated.

2.1 Multistatic Passive Radar

Without loss of generality, the observer will be positioned at the origin. The position of the i th stationary illuminator is given in Cartesian coordinates by $\mathbf{x}_{s,i}$. Time Difference of Arrival (TDoA) and the bistatic Doppler shift are measured, which are directly related to the bistatic range r_i and bistatic range-rate \dot{r}_i , i.e.

$$\begin{aligned} \text{TDoA} &= \frac{r_i - \|\mathbf{x}_{s,i}\|}{c} = \frac{\|\mathbf{p}\| + \|\mathbf{p} - \mathbf{x}_{s,i}\| - \|\mathbf{x}_{s,i}\|}{c}, \\ \text{Doppler} &= -\frac{\dot{r}_i}{\lambda} = -\frac{1}{\lambda} \left(\frac{\mathbf{p}}{\|\mathbf{p}\|} + \frac{\mathbf{p} - \mathbf{x}_{s,i}}{\|\mathbf{p} - \mathbf{x}_{s,i}\|} \right)^T \cdot \mathbf{v}, \end{aligned} \quad (1)$$

where c is the speed of light, λ the wavelength of the RF signal and \mathbf{p} and \mathbf{v} denote the target's position

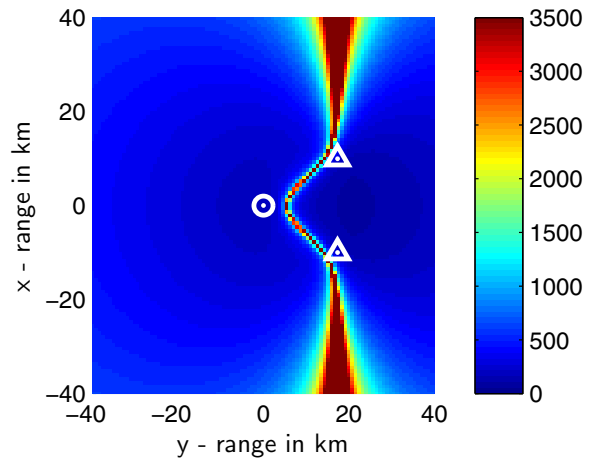
and velocity in Cartesian coordinates. The bistatic range equation describes ellipsoids in three-dimensional Cartesian space with foci at the observer and illuminator position. The bistatic Doppler depends on the target velocity component normal to the surface of the ellipsoid. Considering the target state $\mathbf{x} = (\mathbf{p}, \mathbf{v})^T$ and the measurement vector $\mathbf{z}_i = (r_i, \dot{r}_i)^T$, the measurement equation is in the following abbreviated by $\mathbf{z}_i = h(\mathbf{x}, \mathbf{x}_{s,i})$.

For reason of association complexity [1], we estimate the target position and velocity from a minimum number of two synchronous measurements using two different illuminators ($\mathbf{x}_{s,1}$ and $\mathbf{x}_{s,2}$). In a simplified 2D consideration, this would be intersecting two ellipses with foci at the origin and $\mathbf{x}_{s,i}$. This basically means solving two quadratic equations successively, which can result in up to four solutions; but since in this case the two ellipses share one focal point, there will be only two solutions. Thus, in the 2D scenario and, even more generally, if the target height is exactly known, the target position can be derived analytically.

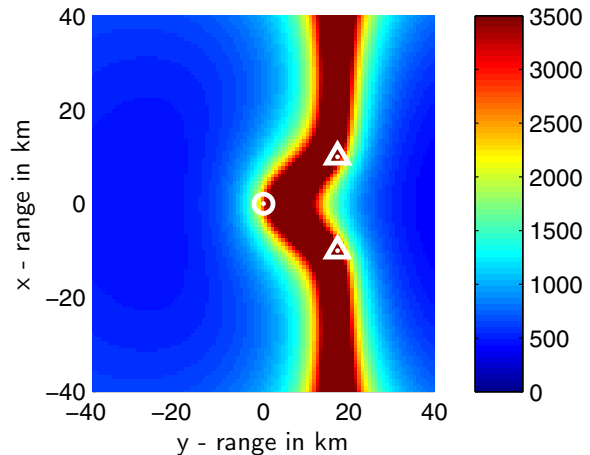
Considering airborne targets the assumption of 2D problem is generally not fulfilled. The unknown target height has influence on the position estimate, which was discussed in more detail in [4]. The calculation of CRLB [4] characterizes the impact of the unknown height on the position estimate. In figure 1 the accuracy of the position estimate as indicated by the CRLB is plotted for each point in a 80 km×80 km region and a fixed constellation of two illuminators and one receiver. Already for the optimistic case of known target height (figure 1(a)), we notice that there are regions of poor estimation performance that enlarge with decreasing a-priori knowledge on target height (see figure 1(b) for a standard deviation (std) of 2km in target height).

In 3D, intersecting two ellipsoids yields as a solution an ellipse in 3D. Assuming some directional information and neglecting heights below zero we can constrain the locations of potential targets (uncertainty region) to an ellipse sector that is plotted as an example in figure 2 for three different targets locations in y - z view. If we additionally constrain the target height to values below, for example, 10 km, we can draw conclusions from the curvature of the ellipse sector on the magnitude of position estimation error. If the target is in the far field of the sensors¹, the influence of the height on the estimate is insignificant. We deduce that, for targets in the far field of the illuminators and receiver, the bistatic range equations holds no information about the target height. For near field considerations it is quite obvious that approximating by a single Gaussian fails; thus we propose a GM approximation for more robust filter initialization.

¹where the range is measured orthogonal to the line between the illuminators, which corresponds to the y -component of the range in the scenario considered here



(a) CRLB (POS), \mathbf{z} known



(b) CRLB (POS), $\mathbf{z} \sim \mathcal{N}(\mathbf{z}; 5\text{km}, (2\text{km})^2)$

Figure 1: CRLB by exploiting bistatic range and range-rate information generated by two illuminators (triangles) and one receiver (circle) at one single time scan. Units of the colorbar are in m.

2.2 Passive emitter localization

We consider the situation of tracking a non-cooperative emitter using TDOA measurements from a pair of sensors. The emitter is assumed to be stationary, while sensors \mathbf{s}_i , $i = 1, 2$ are moving and collecting TDOA measurements over time. For reasons of simplicity, the emitter and sensors are located in the same plane. Therefore a two-dimensional TDOA scenario is studied. One noise free TDOA measurement localizes the emitter on a branch of a hyperbola with the two sensors as foci. The measurement equation

$$h(\mathbf{x}) = \frac{r_1 - r_2}{c}. \quad (2)$$

is nonlinear, where r_i is the distance of the emitter to sensor i , $i = 1, 2$. The TDoA measurement equation depends only on the emitter position, denoted by \mathbf{x} , the sensor positions enter as parameters. The measure-

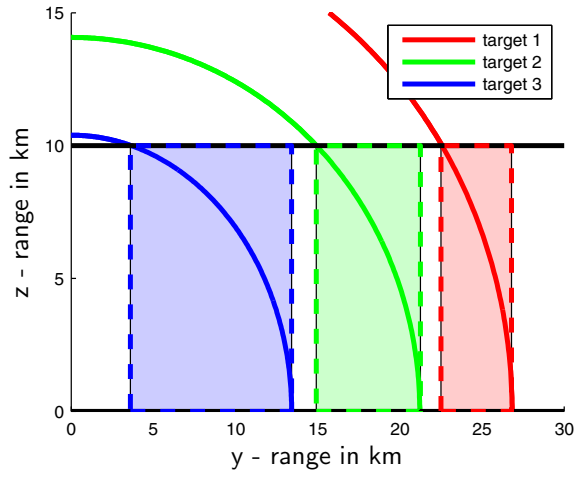


Figure 2: Impact of height uncertainty on Position estimate: estimation ambiguity for three potential targets is plotted in y - z view. The uncertainty can be described by a curve that is bent according to the distance of target, illuminators and receiver. For targets in the near field, the curve is more bent, which means that the inaccuracy in height has a significant impact on the position estimate.

ment function is modeled by adding white Gaussian noise to the measurement equation: $z_t = h(\mathbf{x}) + \nu$. We want to model the likelihood function after the first measurement $p(z_1|\mathbf{x})$ and the posterior density $p(\mathbf{x}|z_1)$. Figure 3 shows the posterior density after the first measurement. The approximation with a single Gaussian

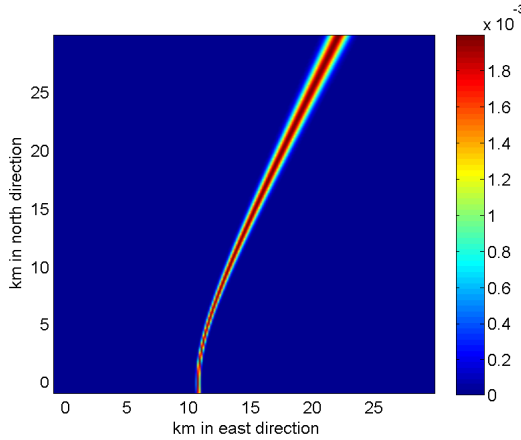


Figure 3: Posterior density after one TDOA measurement

fails. Thus the localization with a single Kalman Filter, EKF or UKF, depends strongly on the initialization and the KF diverges in adverse scenarios, see [5]. Therefore we approximate the density with a GM. For each summand of the GM an EKF is started. Results of a static GM filter are shown in [5]. In the present paper, we introduce an hypothetical azimuth measurement

at a certain range, and obtain a two-dimensional measurement space which we transform into the Cartesian state to find the parameters of the GM.

The need for a GM approximation in the two passive tracking applications is cause of the incomplete measurement information. To determine a Cartesian estimate, prior information about target height or an azimuth can be exploited.

3 Some Fundamentals on GM approximations

In this section we shortly repeat some fundamentals on GMs and derive formulas for passive tracking applications. The GM approximation lemma [3] states that every probability density p can be approximated (as closely as desired) by a GM:

$$p(\mathbf{x}) \approx \sum_{i=1}^n \alpha_i \mathcal{N}(\mathbf{x}; \mathbf{m}_i, \mathbf{B}_i), \quad (3)$$

where n is sufficiently large and the positive scalars α_i , mean vectors \mathbf{m}_i and positive-definite covariance matrices are chosen properly.

In target tracking we need to determine the likelihood $p(\mathbf{z}|\mathbf{x})$, from that we can derive the posterior density according to Bayes rule:

$$p(\mathbf{x}|\mathbf{z}) = \frac{p(\mathbf{z}|\mathbf{x}) \cdot p(\mathbf{x})}{\int p(\mathbf{z}|\mathbf{x}) \cdot p(\mathbf{x}) d\mathbf{x}} \quad (4)$$

Since for track initialization a priori knowledge in \mathbf{x} is not available, $p(\mathbf{x})$ is constant, which means that every possible state of \mathbf{x} is equally likely, i.e.

$$p(\mathbf{x}|\mathbf{z}) = \frac{p(\mathbf{z}|\mathbf{x})}{\int p(\mathbf{z}|\mathbf{x}) d\mathbf{x}}. \quad (5)$$

Thus, approximating $p(\mathbf{x}|\mathbf{z})$ can be transferred to approximating $p(\mathbf{z}|\mathbf{x})$ by a GM. Since this is not a probability density of \mathbf{x} , the Gaussian sum takes the form:

$$p(\mathbf{z}|\mathbf{x}) \approx \sum_{i=1}^m \alpha_i \mathcal{N}(\mathbf{z}; \mathbf{m}_i, \mathbf{B}_i), \quad (6)$$

where $\sum_{i=1}^m \alpha_i = \int p(\mathbf{z}|\mathbf{x}) d\mathbf{x} \neq 1$.

3.1 How to find the parameters of the GM?

Approximating a given probability density by a GM (Eq. 3) means finding the best fit of parameters according to a pre-defined number of summands; thus a multi-dimensional search needs to be performed. The Expectation Maximization (EM) algorithm is quite a popular solution to this problem, see for example [6], due to its simple implementation and its proven convergence to local maxima. However the EM requires a good initialization and its computational complexity

depends on the dimension of \mathbf{x} (Eq. 3). An optimal search is therefore often not applicable for real-time track initialization.

In [3] an approach is presented that requires for the known functional relationship of the measurement and target state, i.e. $\mathbf{z} = h(\mathbf{x}) + \mathbf{w}$, with some noise vector \mathbf{w} , only an approximation of the usually known pdf of \mathbf{w} , i.e. $p_w(\mathbf{z} - h(\mathbf{x}))$. The desired Gaussian sum approximation in Cartesian coordinates can then be deduced from this by transformation. This provides two crucial advantages: First, finding a Gaussian sum approximation of p_w is generally simpler than of the Cartesian pdf. Second, assuming identical pdfs of different measurements the parameter fitting needs to be done only once to derive different types of Cartesian sums.

In our applications the problem is reduced to fitting a uniform density. Following the description in [7] we use the following steps to approximate a uniform density in the interval $[a, b]$:

- choose n mean values μ_i to be equally spaced in the considered interval, i.e. $\mathbf{m}_i = a + (2i - 1)(b - a)/(2n)$
- set the weighting factors $\alpha_i = 1/n$
- select the standard deviation $\sigma_{\mu_i} = \sigma_\mu$ such that the L^1 distance is minimized, i.e.

$$\sigma_\mu = \arg \min_{\sigma} \int_{-\infty}^{\infty} \left| \frac{1}{b-a} - \sum_{i=1}^n \alpha_i \mathcal{N}(x; \mu_i, \sigma^2) \right| dx \quad (7)$$

Thus, only a one-dimensional search is required.

A given GM approximation of p_w , results in the following approximation of $p(\mathbf{z}|\mathbf{x})$:

$$\begin{aligned} p(\mathbf{z}|\mathbf{x}) &= p_w(\mathbf{z} - h(\mathbf{x})) \\ &\approx \sum_{i=1}^N \tilde{\alpha}_i \mathcal{N}(\mathbf{z} - h(\mathbf{x}); \tilde{\mathbf{m}}_i, \tilde{\mathbf{B}}_i) \\ &= \sum_{i=1}^N \tilde{\alpha}_i \mathcal{N}(h(\mathbf{x}); \tilde{\mathbf{m}}_i, \tilde{\mathbf{B}}_i) \end{aligned} \quad (8)$$

To approximate equation (6) by means of a GM dependent on the Cartesian state vector \mathbf{x} we follow the derivation in [3]:

$$\begin{aligned} &\mathcal{N}(h(\mathbf{x}); \tilde{\mathbf{m}}_i, \tilde{\mathbf{B}}_i) \\ &\approx \frac{1}{|2\pi\tilde{\mathbf{B}}_i|^{1/2}} e^{-(1/2)(\tilde{\mathbf{m}}_i - h(\mathbf{x}))^T \tilde{\mathbf{B}}_i^{-1} (\tilde{\mathbf{m}}_i - h(\mathbf{x}))} \\ &= \frac{1}{|2\pi\tilde{\mathbf{B}}_i|^{1/2}} e^{-(1/2)(\mathbf{x} - \mathbf{m}_i)^T \mathbf{B}_i^{-1} (\mathbf{x} - \mathbf{m}_i)}, \end{aligned} \quad (9)$$

where $\mathbf{m}_i = h^{-1}(\tilde{\mathbf{m}}_i)$, $\mathbf{B}_i = [\mathbf{J}_h(\mathbf{m}_i)^T \tilde{\mathbf{B}}_i^{-1} \mathbf{J}_h(\mathbf{m}_i)]^{-1}$ and $\mathbf{J}_h(\mathbf{m}_i)$ is the Jacobi matrix with respect to h . Calculation holds only if h is invertible, which also implies that $\dim(\mathbf{x}) = \dim(\mathbf{z})$.

Thus, the Gaussian pdf can be approximated by a pdf on \mathbf{x} and an adequate weight factor:

$$\begin{aligned} &\mathcal{N}(h(\mathbf{x}); \tilde{\mathbf{m}}_i, \tilde{\mathbf{B}}_i) \\ &\approx \frac{|2\pi\mathbf{B}_i|^{1/2}}{|2\pi\tilde{\mathbf{B}}_i|^{1/2}} \mathcal{N}(\mathbf{x}; \mathbf{m}_i, \mathbf{B}_i) \\ &= |\mathbf{J}_h(\mathbf{m}_i)| \mathcal{N}(\mathbf{x}; \mathbf{m}_i, \mathbf{B}_i), \end{aligned} \quad (10)$$

The inverse function theorem states that

$$\mathbf{J}_h(\mathbf{m}_i) = \mathbf{J}_h(h^{-1}(\tilde{\mathbf{m}}_i)) = (\mathbf{J}_{h^{-1}}(\tilde{\mathbf{m}}_i))^{-1} \quad (11)$$

Thus:

$$\begin{aligned} \mathbf{B}_i &= [\mathbf{J}_h(\mathbf{m}_i)^T \tilde{\mathbf{B}}_i^{-1} \mathbf{J}_h(\mathbf{m}_i)]^{-1} \\ &= [\mathbf{J}_{h^{-1}}(\tilde{\mathbf{m}}_i) \tilde{\mathbf{B}}_i \mathbf{J}_{h^{-1}}(\tilde{\mathbf{m}}_i)^T] \end{aligned} \quad (12)$$

In summary:

The desired GM approximation in Cartesian coordinates is given by

$$p(\mathbf{z}|\mathbf{x}) = \sum_{i=1}^N \alpha_i \mathcal{N}(\mathbf{x}; \mathbf{m}_i, \mathbf{B}_i), \quad (13)$$

with weights $\alpha_i = \tilde{\alpha}_i |(\mathbf{J}_{h^{-1}}(\tilde{\mathbf{m}}_i))^{-1}|$, the means $\mathbf{m}_i = h^{-1}(\tilde{\mathbf{m}}_i)$ and covariances $\mathbf{B}_i = [\mathbf{J}_{h^{-1}}(\tilde{\mathbf{m}}_i) \tilde{\mathbf{B}}_i \mathbf{J}_{h^{-1}}(\tilde{\mathbf{m}}_i)^T]$. Note that the mean and covariance of the transformed density match with the results of transforming $\mathcal{N}(\mathbf{z}; \tilde{\mathbf{m}}_i, \tilde{\mathbf{B}}_i)$ by linearization according to h^{-1} .

4 Application of the GM for Track Initialization

This section provides GM approximation results.

4.1 Multistatic Passive Radar: GM Approach for Unknown Target Heights

We model the target height to be uniformly distributed in a suitable large interval $[a, b]$, for example $[0 \text{ km}, 10 \text{ km}]$, and approximate this probability density by a GM. Following the steps for approximating a uniform density described above we calculate the Gaussian summands for $n = 6, 10, 20$ and 49 , results are shown in figure 4(a). Obviously, the GM approximation converges to the true density for increasing N . The approximation is worst in vicinity of the boundaries of the interval; this seems to be admissible in our application and could be counteracted by enlarging the interval.

The std (chosen by the one-dimensional search procedure) corresponding to $N \in \{6, 10, 20, 49\}$ are 799, 522, 282 and 124 respectively. Choosing 10 mixtures seem to give an appropriate approximation of the true density and correspond to a height error about 500 m of each component of the sum. Utilizing the GM

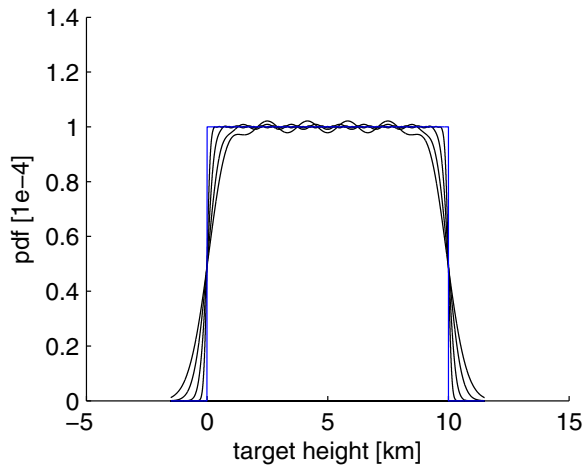


Figure 4: GM approximations of uniform target height for different number of mixtures $N \in \{6, 10, 20, 49\}$

approximation of the target height results in (compare [7]):

$$\begin{aligned}
 p_w(\mathbf{z}^{(a)}|\mathbf{x}) &\approx \sum_{i=1}^N \alpha_i \mathcal{N}(h(\mathbf{x}); \mathbf{z}_i^{(a)}, \mathbf{P}_i^{(a)}) \\
 &\approx \sum_{i=1}^N \beta_i \mathcal{N}(\mathbf{x}; \mathbf{y}_i^{(a)}, \mathbf{Q}_i^{(a)})
 \end{aligned} \tag{14}$$

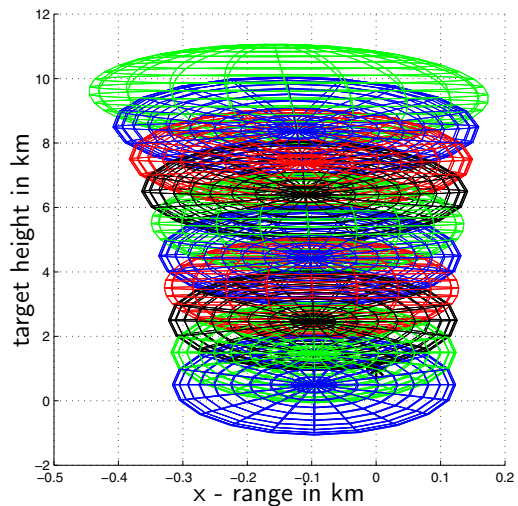
where $\mathbf{z}^{(a)} = (r_1, r_2, \dot{r}_1, \dot{r}_2, h_t, \dot{h}_t)^T$ and $\mathbf{P}_i^{(a)} = \text{diag}(\sigma_r, \sigma_r, \sigma_{\dot{r}}, \sigma_{\dot{r}}, \sigma_{h_t}, \sigma_{\dot{h}_t})$, $\beta_i = \alpha_i \cdot \sqrt{\det(\mathbf{Q}_i)}$ and \mathbf{y}_i and \mathbf{Q}_i are the results of the transformation from measurements into Cartesian (either by Unscented Transform (UT) or Linearization). It follows that:

$$p(\mathbf{x}|\mathbf{z}) \approx \sum_{i=1}^N \tilde{\beta}_i \mathcal{N}(\mathbf{x}; \mathbf{y}_i^{(a)}, \mathbf{Q}_i^{(a)}), \tag{15}$$

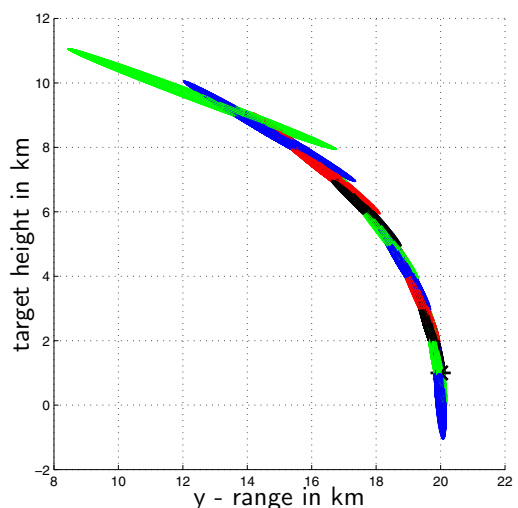
where $\tilde{\beta}_i = \beta_i / (\sum_{i=1}^N \beta_i)$. Results for 10 mixture components and a specific multistatic geometry are shown in figure 5. Two sources are placed at $(-10 \text{ km}, \sqrt{3} \cdot 10 \text{ km}, 0 \text{ km})^T$ and $(10 \text{ km}, \sqrt{3} \cdot 10 \text{ km}, 0 \text{ km})^T$; the target is located at $(0 \text{ km}, 20 \text{ km}, 1 \text{ km})$ and the receiver is placed in the origin. As seen in the picture, the Cartesian covariances increase with increasing height; thus also the probabilities of the respective Gaussians increase.

Simulation Results

In simulation we consider a $80 \text{ km} \times 80 \text{ km}$ region, see figure 6, and for each point in a grid we run 100 Monte-Carlo Runs and simulate measurements in bistatic range (std $\sigma_r = 100 \text{ m}$) and bistatic range-rate (std $\sigma_{\dot{r}} = 1 \text{ m/s}$) for two illuminators (shown by triangles) and a single receiver (circle). The target height is sampled from a uniform distribution in $[0 \text{ km}, 10 \text{ km}]$, and velocity from a Gaussian density with $\dot{x}, \dot{y} \sim \mathcal{N}(\mathbf{0}, 100 \text{ m/s}^2)$; velocity in height is assumed to



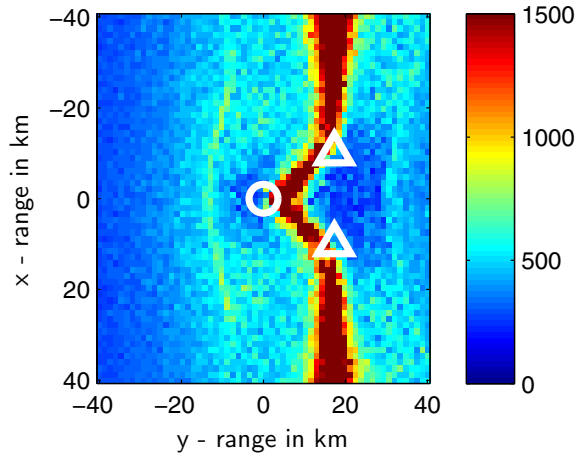
(a) Approximated Cartesian density (x-z view)



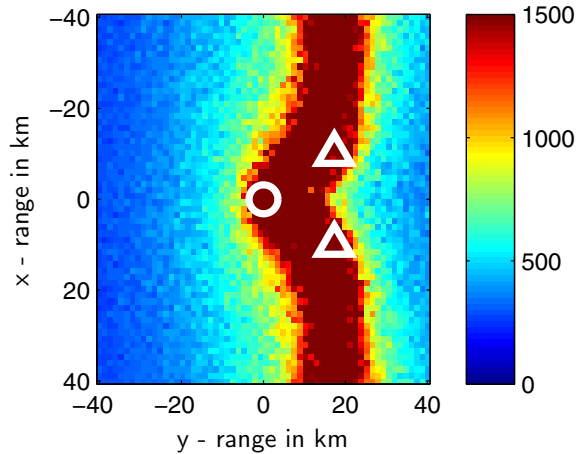
(b) Approximated Cartesian density (y-z view)

Figure 5: GM approximations of 3D Cartesian position. Note the different scaling in (a) and (b).

be zero. At the first time scan, we generate a Gaussian sum with 10 components as explained above and start a bank of UKFs from these initialization points. The Root Mean Squared Error (RMSE) is calculated for the “best” result (according to the track score) after a filtering period of 60 seconds. Results for position, and height estimates are provided in figure 6(a), and figure 7(a). The Gaussian sum approach is compared to the initialization by a single Gaussian, see figure 6(b), and figure 7(b). The results show that with the GM approximation, filter divergence due to bad initialization can be prevented in some regions. Initialization is worse, even for the GM approach, for decreasing range, measured orthogonal to the line of illuminator and re-



(a) RMSE (POS), Gaussian Sum Initialization



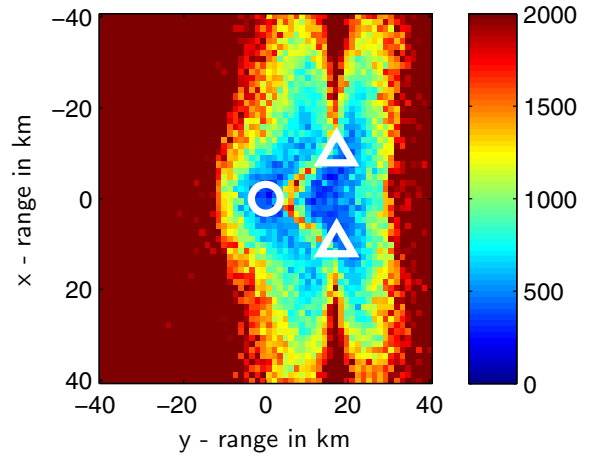
(b) RMSE (POS), Single Gaussian Initialization

Figure 6: Position estimation error of tracking result.

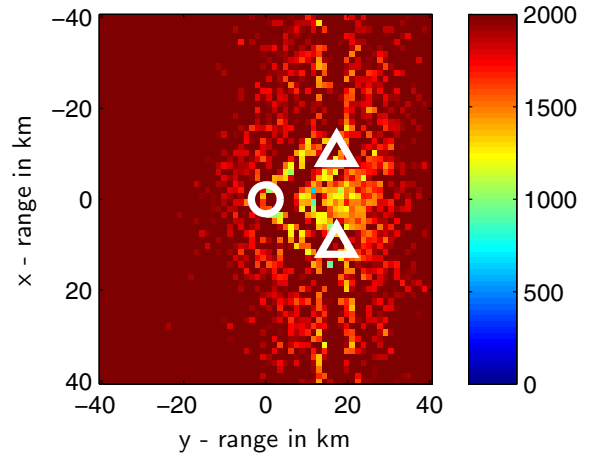
ceiver. However, the target height can best be estimated in those regions where position estimation is difficult. Since for some regions of the observation area the approximation by a GM does not give any advantage but results in increased complexity, we implemented an adaptive scheme that uses the GM approximation only if, in the case of approximation by a single Gaussian, the size of the Cartesian covariance matrix in x - y position exceeds 1000 m. The transition region between single Gaussian and GM initialization becomes apparent in figure 6(a) by a small area of increased RMSE, which is improved again when the target is nearer to illuminators and receiver, which means that the GM approximation starts.

4.2 Passive emitter localization: GM Approximation of the TDOA hyperbolae

By introducing a hypothetical azimuth measurement we obtain a two-dimensional measurement space, which



(a) RMSE (POS), Gaussian Sum Initialization



(b) RMSE (POS), Single Gaussian Initialization

Figure 7: Height estimation error of tracking result.

we can transform into the two-dimensional Cartesian state space. The hypothetical azimuth measurement is generated from one of the sensor positions or an appropriate reference point. Now, we can use the above described approximation algorithm. We compare this algorithm with a proposed sum approximation of Mušicki in [8], which will be referred to as M approximation in the following. The M approximation was originally used within a GM filter [8]. In our implementation it is used for track initialization only. It consists of the following steps:

- The $\pm\sigma$ hyperbolae are transformed into the Cartesian observation space. They are represented by the $\pm\sigma$ hyperbolae.
- The area which the $\pm\sigma$ hyperbolae enclose is inscribed by a sum of ellipses: the same number of points is chosen on the $\pm\sigma$ hyperbolae.
- An ellipse is inscribed in the quadrangle formed by one pair of points on the $+\sigma$ hyperbolae and one

pair of points on the $-\sigma$ hyperbolae

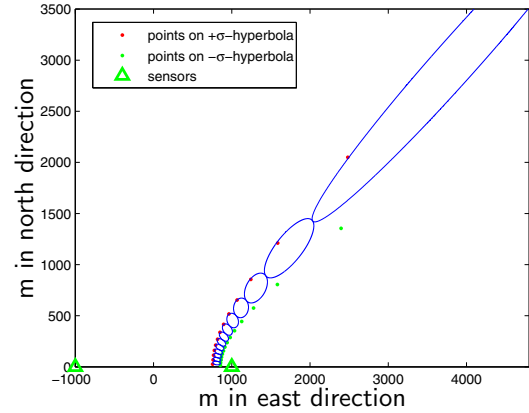
- The ellipse is described by the covariance with the centre at the mean. The weight of the summand is given by the square root of the determinant of covariance.

For the M approximation the hypothetic azimuth measurements are generated from the nearest sensor position. The approximation is shown in figure 8(a). The tube which the \pm hyperbolae form is filled with the Gaussian terms, the ellipses. Notice that the size of the Cartesian covariances of the components of the GM differs significantly. The EKF as well as the UKF do not only suffer from inconsistent initialization, but very large covariances are also a problem (the reason why the initialization covariance will not be chosen arbitrarily large). From an initialization point of view the covariances should be chosen to have more or less the same size. Therefore it may be an important issue to choose the point from which the hypothetic azimuth measurement is generated appropriately in order to make the approximation more robust. To generate Gaussian terms with equal covariances we propose a robust approach of GM approximation. Without loss of generality we place the two sensors at the x -axis with the center at the origin. Then, we limit the extension of the branch of the hyperbola according to the maximal coverage of the sensors and the look direction of the sensors. The reference point (from which the azimuth directions are generated) is chosen to be in equal distance to the intersection point of the hyperbola with the x -axis and the maximal coverage region. The hypothetic azimuth measurements are assumed to be uniformly distributed in a suitable interval, for example $[0^\circ, 60^\circ]$. The intersection points of the external bearing lines and the reference point should form nearly an equilateral triangle with the maximal range as lateral range. As described above, we approximate the azimuth interval with a Gaussian sum. We get uniformly distributed azimuth angles and the associated angle standard deviation σ_α . Using the measurement set $\mathbf{z}^{(a)} = (z_t, \alpha)$ we obtain

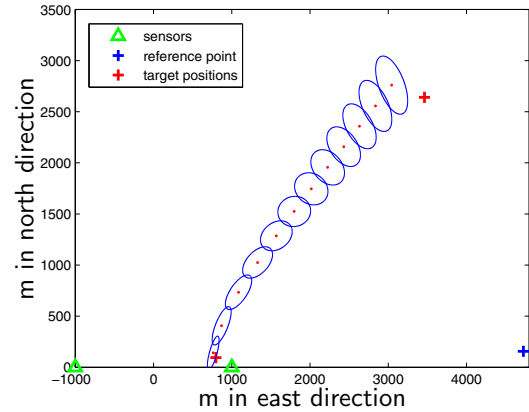
$$\begin{aligned}
 p_w(\mathbf{z}^{(a)}|\mathbf{x}) &\approx \sum_{i=1}^N \alpha_i \mathcal{N}(h(\mathbf{x}); \mathbf{z}_i^{(a)}, \mathbf{P}_i^{(a)}) \\
 &\approx \sum_{i=1}^N \beta_i \mathcal{N}(\mathbf{x}; \mathbf{y}_i^{(a)}, \mathbf{Q}_i^{(a)})
 \end{aligned} \tag{16}$$

where $\mathbf{P}_i^{(a)} = \text{diag}(\sigma_t, \sigma_\alpha)$, $\beta_i = \alpha_i \cdot \sqrt{\det(\mathbf{Q}_i)}$ and \mathbf{y}_i and \mathbf{Q}_i are the results of the transformation from measurements into Cartesians by UT. From this the posterior density $p(\mathbf{x}|\mathbf{z})$ is derived, (see equation 15). Figure 8(b) shows the GM for the robust approach. The reference point and the two external points of the hyperbolae form a nearly equilateral triangle, the means

of the Gaussian terms are relatively equally distributed. The centers of the ellipses are the means, the ellipses the covariances and the square root of the determinant the weights of the Gaussian summands.



(a) M approximation



(b) robust approximation

Figure 8: Gaussian mixture approximation of a TDOA measurement

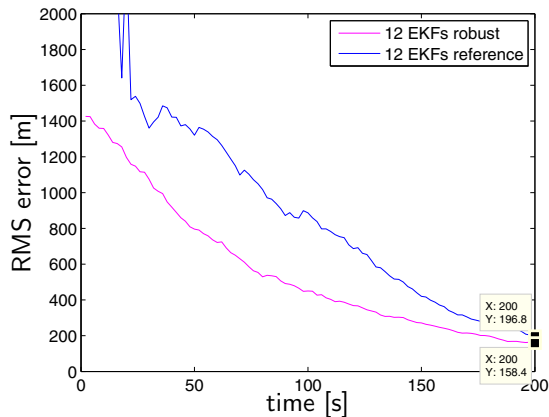
Simulation Results

The following scenario is considered to compare the performance of the two GM approaches: sensors, located at $(-1000 \text{ m}, 0 \text{ m})$ and $(1000 \text{ m}, 0 \text{ m})$ move in east direction at a constant speed of 10 m/s . The emitter is assumed to be stationary. Several target positions are chosen on the hyperbolae of figure 8 and for each point a Monte Carlo Simulation is started. The results presented here are based on 100 measurements averaged over 100 independent Monte Carlo simulations with a sampling interval of two seconds. The measurement standard deviation in the range domain is assumed to be 100 m .

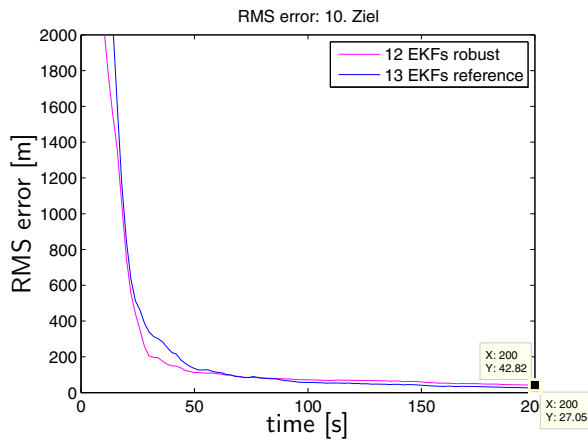
The GM is used to initialize a bank of EKFs. The results are shown in figure 9. In the far field of the sensors, the robust approach shows better results as the

M approximation. In the region near to the sensors, where the M approximation has more little Gaussian terms, its performance is a little better than the robust approach.

The two GM approximation techniques differ accord-



(a) remote



(b) near

Figure 9: Comparison of the robust and M approximation

ing to two aspects: the method of generating the Cartesian ellipses and the choice of the reference point. Responsible for their different performance seems to be the choice of the reference point in our application.

5 Conclusions

In this paper we investigate the GM approach to initialize passive tracking filters. An algorithm for finding the parameters of the Gaussian sum is derived, which can be applied to different tracking applications. The advantage of the Gaussian sum approach is shown by comparison with existing techniques. The GM approach has many applications, in the multistatic application, it could also be considered for handling estimation from bistatic measurements of the bistatic range and azimuth. This is in compliance with the usually

poor angular resolution (if available) capabilities of the considered sensors.

References

- [1] M. Daun and W. Koch, “Multistatic target tracking for non-cooperative illumination by DAB/DVB-T,” in *Proc. IEEE Radar Conference RADAR '08*, 26–30 May 2008, pp. 1–6.
- [2] M. Daun and F. Ehlers, “Tracking Algorithms for Bistatic Sonar Systems,” in *Informatik 2009, Workshop Sensor Data Fusion: Trends, Solutions, Applications*, Lübeck, Germany, October 2009, pp. 2283–2295.
- [3] W. I. Tam, K. Plataniotis, and D. Hatzinakos, “An adaptive Gaussian sum algorithm for radar tracking,” *Signal Processing*, vol. 77, pp. 85–104, 1999.
- [4] M. Daun and C. R. Berger, “Track initiation in a multistatic DAB/DVB-T network,” in *Proc. 11th International Conference on Information Fusion*, 2008, pp. 1–8.
- [5] R. Kaune, “Gaussian Mixture (GM) Passive Localization using Time Difference of Arrival (TDOA),” in *Informatik 2009, Workshop Sensor Data Fusion: Trends, Solutions, Applications*, Lübeck, Germany, October 2009, pp. 2375–2381.
- [6] N. Vlassis and A. Likas, “A Greedy EM Algorithm for Gaussian Mixture Learning,” *Neural Processing Letters* 15, pp. 77–87, 2002.
- [7] H. Sorenson and D. Alspach, “Recursive Bayesian estimation using Gaussian sums,” *Automatica*, vol. 7, pp. 465–479, 1971.
- [8] D. Mušicki and W. Koch, “Geolocation using TDOA and FDOA measurements,” in *Proc. 11th International Conference on Information Fusion*, 2008, pp. 1–8.